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METHOD AND APPARATUS FOR EXPERIMENTAL DETERMINATION OF HEAT EXCHANGE IN SOIL

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METHOD AND APPARATUS FOR EXPERIMENTAL DETERMINATION OF HEAT EXCHANGE IN SOIL

C. P. Bespalov

ABSTRACT: The article is a generalization of experimental and theoretical studies dealing with the development and improvement of methods of determining heat flux in soil.

The quantitative measurement of heat flux between the surface and deep /68* layers of soil is an important practical problem whose solution can be used as a basis for developing necessary methods of calculation and regulation of heat supplies in the soil. In addition, heat exchange between the surface and deep layers of the soil, as one of the terms of the equation of heat balance of an actual surface, plays an important role in the determination of those components of thermal balance whose direct measurement poses certain practical difficulties (turbulent heat exchange and heat losses through evaporation). Heat flux from deep layers to the surface of the soil is the principal heat source reaching the surface at night and must be taken into account in forecasting the nocturnal temperature drop both of the layer of the air nearest the surface and of the surface of the soil (radiation fogs, frosts). On the basis of a calculation of the consideration of the numerous applications of quantitative characteristics of heat exchange in the soil to the development of methods and apparatus for their determination, a great deal of work has been done both in the Soviet and foreign literature.

Until recently, the method of calculating heat exchange between the surface and deep layers of the soil, based on the equation of molecular thermal conductivity, took the direction of increasing accuracy and reliability of determination of temperature gradients in the soil and the thermophysical characteristics of the latter (primarily thermoconductivity). Owing to the technical complexity of determining the temperature gradient in the soil at the surface, the heat flux from the surface into the depths of the soil (or

*Numbers in the margin indicate pagination in the foreign text.

vice versa) is considered as the sum of the heat flux at a depth z_1 and the change in heat content of a layer of soil from the surface to this depth. The depth z_1 is selected so that the thermal regime of the soil at this depth will be close to quasistationary. Then the vertical temperature gradient $\partial T/\partial z$ can be replaced by the reduced temperature differential at two depths, i.e., $(T_1 - T_2)/(z_1 - z_2)$. The depth at which the thermal regime of the soil may be considered quasistationary is usually 5-15 cm; the change in heat content of the upper layer is usually disregarded. However, in many practically /69 important cases failure to consider the change in heat content of the upper layer of soil may lead to significant errors.

To measure the soil temperature at depths up to 20 cm at hydrometeorological stations, as well as in the majority of investigative tasks, the so-called elbow thermometers of Savinov are used. However, these thermometers are very crude and, what is most important, do not provide the necessary temperature measurement accuracy (the actual precision of the temperature measurement provided by these thermometers is no more than 0.3°). In addition, these thermometers can only be used during the warm season in unfrozen soil [1]. Hence, in order to increase the measurement accuracy of heat flux in the soil, it is important to develop more sophisticated methods of measuring the temperature distribution in the soil.

As we mentioned earlier, the heat flux from the surface into the depths of the soil is determined by the equation of molecular thermoconductivity

$$Q = -\int_0^{\tau} \lambda \left.\frac{\partial T}{\partial z}\right|_{z=0} d\tau = -\int_0^{\tau}\int_0^{z_1} c\rho T(z, \tau)\, dz\, d\tau - \int_0^{\tau} \lambda \left.\frac{\partial T}{\partial z}\right|_{z=z_1} d\tau, \qquad (1)$$

where τ is the time interval for which the heat flux is calculated, λ is the thermal conductivity of the soil, $c\rho$ is the volume thermal capacity (ρ = density, c = specific thermal capacity). Therefore, in addition to measuring the temperature distribution in the soil, calculating the heat flux requires knowledge of the thermophysical characteristics of the soil (λ and $c\rho$). Development of methods of calculating thermal exchange between the surface and deep layers of soil at the Main Geophysical Observatory has been conducted primarily with an eye toward direct determination of heat flux from temperature distribution in the soil. The thermal conductivity k of the soil is then determined from the observed natural temperature pattern in the depths, while the thermal conductivity is found as its derivative over the volume of thermal capacity ($\lambda = kc\rho$). Determination of the volume thermal capacity of the soil is based on the known (tabular) thermal capacity of the solid skeleton of the soil and its dependence on soil moisture. In this connection, thanks to the work of D. L. Laykhtman [2, 3], G. Kh. Tseytin [4, 5] and others, a number of important results have been obtained which make it possible in many instances to calculate with sufficient accuracy the heat flux from the surface into the depth of the soil for time intervals of two to four hours or more, as well as on the basis of a calculation of the mean effective values of the thermo-physical characteristics to determine the heat flux for still shorter time intervals.

One of the most significant shortcomings of this method is the fact that the thermoconductivity values must be determined indirectly, through the temperature conductivity in volume thermal capacity; however, the determination of the latter with a sufficient degree of accuracy poses considerable technical difficulties even for laboratory conditions. Determination of temperature conductivity from a sequence of observed temperature distributions in the soil also poses certain difficulties and is not always possible.

Development of direct methods of determining thermophysical characteristics (thermal conductivity, temperature conductivity and thermal capacity) of various materials essentially began at the same time as the development of the theory of thermal conductivity. Papers in this field, beginning with Bio and Fourier, continue to be published at the present time, developing and improving over almost half a century.

The specific difficulty of this problem lies in the broad range of changes in the thermophysical characteristics, as well as the high degree of diversity /70 of conditions under which they are determined (size, sample shape, structure of the material, admissible heating, etc.).

Without pausing here to go into a discussion of the extensive literature devoted to methods of determining thermophysical characteristics of various materials, we will merely mention that methods of stationary thermal flux are the ones that have undergone the greatest development for diverse technical purposes thus far; this happens when a stationary heat flux of a known power passes through a sample of material and the temperature gradient in the sample is measured (usually the difference in temperature at the limits of the sample); on the basis of these data, the thermal conductivity of the sample is calculated. A number of technical complications in the use of the method of stationary thermal flux [6] has led to the development of new methods, so-called methods of a regular regime. Developed by Professor G. M. Kondrat'yev [7] and his students, these methods are based on the relationship between the cooling rate, heat loss and thermophysical characteristics of the sample (its shape and size are also taken into account). These methods, when carried out with sufficient care, allow an accuracy of determination of thermal conductivity and temperature conductivity of up to 5% and are widely used in technology. They have a significant shortcoming with respect to their use, namely, the need to remove samples for testing and the performance of measurements under laboratory conditions in special calorimeters, which does not permit them to be used without significant changes under field conditions.

Nevertheless, the principal conditions of a regular regime are of definite interest and may be used in working out probe methods by discovering the relationships between heat loss and thermophysical characteristics of the medium.

Probe methods [6] are of the greatest interest for determining thermophysical characteristics of soil under field conditions. They are based on a situation in which some heat source of a certain power is located in the soil, under whose influence the temperature of the medium near the source begins to change. From the solution of the thermal conductivity equation it is possible by measuring the temperature change at a point located a known distance from the heat source to determine the thermal conductivity and the temperature conductivity of the medium. Such a probe was developed by A. F. Chudnovskiy [6], M. A. Kaganov [11], D. L. Laykhtman [8] and other specialists, both Soviet and foreign, who have suggested a number of different designs and methods of measurement. Probe methods, which make it possible at any time to determine the value of the thermophysical characteristics of the soil, make it possible to calculate the heat flux for any time interval directly using formula (1) and do not require long time intervals of observation of the natural temperature pattern in the soil at some depth in order to determine its thermophysical characteristics.

In addition, probe methods give the values of the thermophysical characteristics of the soil at the depth of installation of the probe, and not average effective values for a layer of significant thickness. This makes it possible to calculate with considerable accuracy the heat flux according to formula (1). From a great many designs of probes for determining thermophysical characteristics of the soil, we selected two of the most strictly theoretically based ones, which are also widely used.

First of all, let us discuss the probe developed by D. L. Laykhtman [8]. This probe is a linear heat source in the form of a tensed string, located a fixed distance r from one of the junctions of a differential thermocouple (the thermocouple is also made in the form of a wire wound parallel to a heater). /71
A continuous pulse of current is passed through the heater (for ~60 to 120 sec). At a distance from the heater, the time required to achieve maximum temperature increase and the value of this maximum increase are measured. On the basis of these data and the amount of heat emitted at the heater, one can calculate the thermal conductivity and the temperature conductivity of the medium in which the probe is located. The calculations are performed on the basis of an exact solution of the equation of thermal conductivity and with the use of nomograms compiled by L. A. Klyuchnikova, which posed no difficulties of any kind for an observer with average training.

The shortcomings of the probe include inconvenience of installation of the soil (it is necessary to keep a distance between the heater and the thermocouple, which frequently poses certain difficulties, as well as the unreliability of the thermal contact of the heater and thermocouple with the soil. The latter is very important, since the soil particles are usually greater than the diameter of the heater. This is the main reason for the significant errors that have been obtained in measurements using this "heat needle" [9].

A heat probe with a linear heat source was developed for determining thermophysical characteristics of snow and was used by us under expedition conditions at the "North Pole 4" and "North Pole 5" stations [10]. With some slight changes in design characteristics, this probe has also been used for measuring thermophysical characteristics of ice.

The second probe which is widely used especially in the work of the Agrophysical Institute of the All-Union Academy of Agricultural Sciences im. V. I. Lenin, is the spherical probe of Kaganov and Chudnovskiy [11]. The probe is a thin-walled hollow sphere 2 cm in diameter. There is a coil located at the center of the sphere for heating it by electricity, and the junction of a differential thermocouple is attached to the inside wall; the second junction of the thermocouple is extended outside and located in the soil at a short distance from the probe (where it will not be affected by the heating of the probe). An electrocurrent of constant power is passed through the coil and the temperature pattern at the surface of the sphere is measured. With a number of simplifying assumptions (thermal capacity of the probe equal to zero, etc.), the solution of the equation of thermal conductivity gives the following expression for the temperature change of the sphere with time:

$$T(\tau) = \frac{q}{4\pi\lambda r}\left[1 - \Phi\left(\frac{r}{2\sqrt{k\tau}}\right)\right], \qquad (2)$$

where $q = 0.24I^2R$ is the power of the heat source at the center of the sphere (I is the intensity of the current, R is the resistance of the spiral), r is the radius of the sphere, λ is the thermal conductivity of the soil, k is its temperature conductivity, $\Phi(u)$ is the probability integral. If in (2) we expand $\Phi(r/2\sqrt{k\tau})$ in a series and limit ourselves to the first term of the expansion, we will obtain the following approximate formula for $T_{(\tau)}$

$$T(\tau) \simeq \frac{q}{4\pi\lambda r}\left(1 - \frac{r}{\sqrt{k\tau}}\right). \qquad (3)$$

As we can see from (3), $T(\tau)$ is a linear function of $1/\sqrt{\tau}$ and

$$\lim_{\tau\to\infty} T(\tau) = \frac{q}{4\pi\lambda r}. \qquad (4)$$

Having determined $T(\tau)_{\tau} = \infty$, we can easily calculate λ from (4). To determine $T(\tau)_{\tau} = \infty$, Kaganov and Chudnovskiy suggested a graphic method con- /72
sisting in the fact that measured values of T at fixed moments of time are used to plot a graph showing the dependence of T on $1/\sqrt{\tau}$, which then [according to (3)] is extrapolated linearly to $1/\sqrt{\tau} = 0$ $(\tau = \infty)$.

Despite its simplicity, this method has a number of shortcomings that complicate its use. Due to the approximate nature of formula (3) a relationship which is nearly linear is only obtained at sufficiently high values of τ, which requires considerable time for measurement (according to the instructions, approximately an hour). With a recommended current of 0.5 A, this induces a significant drying out of the soil around the probe shell. The result of the drying of the soil is quite clearly evident in Figure 1, where the values of T are plotted as a function of $1/\sqrt{\tau}$ for moments τ = 2, 3, 4, 6 min. at a current of 0.1 A, which is considerably below the recommended measurement regimes both with respect to the current and the measurement time (it was necessary to use highly sensitive galvanometers for measuring T). Only the drying ~~of the soil (which consumes a portion of the heat emitted by the probe) can~~ explain instances of a decrease in the temperature of the surface of the probe with time while according to (3) the temperature of the probe is supposed to increase with time. Consideration of the influence of drying of the soil in determining its thermal physical characteristics by means of the spherical probe of Kaganov and Chudnovskiy, as well as the influence of disruption of the soil structure in inserting the probe was discussed in a paper by V. P. Deryabin [12]. However, the practical utilization of this method is difficult due to the necessity of an additional determination of the volume thermal capacity of the soil by an independent method.

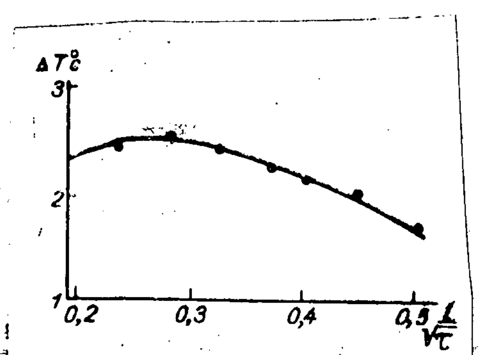

Figure 1. Surface Temperature Pattern of a Spherical Probe.

It should be pointed out that Chudnovskiy obtained the solution of the equation of thermal conductivity for a spherical probe in a form different from (2), and this led to the unsuitable method of calculating thermal conductivity. Chudnovskiy's solution may be converted to the form of (2) by replacing the variable in the probability integral. If we use the solution of the equation of thermal conductivity for a spherical probe in the form (2), we can easily achieve a significant reduction in measurement time (to 5-6 min.); in addition, we then determine simultaneously not only the thermal conductivity of the soil but also the temperature conductivity. To do this, it is sufficient to measure the temperature increase of the surface of the sphere for two moments in time τ_1 and τ_2. From the ratio $T(\tau_1)/T(\tau_2)$ we can determine k

$$\frac{T(\tau_1)}{T(\tau_2)} = \frac{1-\Phi\left(\frac{r}{2\sqrt{k\tau_1}}\right)}{1-\Phi\left(\frac{r}{2\sqrt{k\tau_2}}\right)}. \tag{5}$$

After determining k, we calculate λ directly from (2). In calculating λ from (2) we can use measured values such as $T(\tau_1)$ and $T(\tau_2)$. The difference /73
between the values λ_1 and λ_2 obtained on the basis of the values of $T(\tau_1)$ and $T(\tau_2)$ will characterize the error in the method of determining λ.

To simplify the technique of calculation, we can use the graphs of the functions which were plotted earlier

$$\frac{1-\Phi\left(\frac{r}{2\sqrt{k\tau_1}}\right)}{1-\Phi\left(\frac{r}{2\sqrt{k\tau_2}}\right)} = \Psi(k),$$

$$1-\Phi\left(\frac{r}{2\sqrt{k\tau_1}}\right)$$

and

$$1-\Phi\left(\frac{r}{2\sqrt{k\tau_2}}\right)$$

for the previously selected values of τ_1 and τ_2 for a given probe radius.

The calculations of thermal conductivity and temperature conductivity of the soil calculated by this method on the basis of measurement during the expedition of the main geophysical observatory in northern Kazakhstan (1955) showed satisfactory agreement between λ_1 and λ_2: the differences, as a rule, were noted in the third and fourth significant figures (see Table 1). To a certain extent, the good agreement of the results was explained by an improvement in the measurement apparatus employed, particularly by the rather high-sensitivity and low-inertia galvanometers (GZP-47 and loop).

The relationships obtained may be used for selecting the basic parameters of the probe, primarily its dimensions. As we can see from Figure 2, the dimensions of the Kaganov-Chudnovskiy probe for the values of temperature conductivity which are usually encountered, $k \cong 4.5 \cdot 10^{-3}$ cm^2/sec, are insufficient: to obtain reliable values of k the ratio $T(\tau_1)/T(\tau_2)$ must be determined with a high degree of accuracy (at least up to 0.01°), which presents certain technical difficulties. However, by increasing the radius of the probe to r = 2 cm, the accuracy with which k (and consequently λ) are determined may /74 be increased significantly (with the same accuracy of measurement of $T(\tau_1)/T(\tau_2)$). As we can see from (5), the decrease in τ_1 and τ_2 also corresponds to an increase in r. However, the decrease of τ_1 and τ_2 in the first place is less effective due to the fact that in (5) they are introduced in the power 1/2, and in the second place, undesirably, inasmuch as this reduces the values of $T(\tau_1)$ and $T(\tau_2)$ and increases the nonuniformity of the initial distribution of the temperature within the body of the probe.

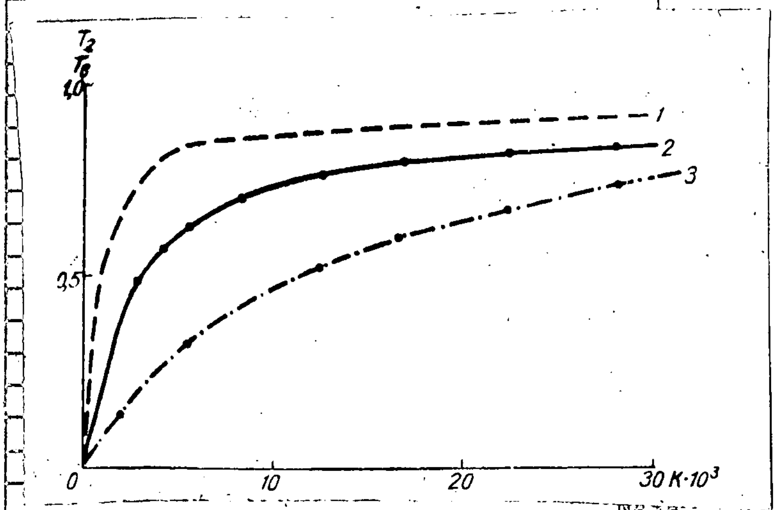

Figure 2. Ratio T_2/T_6 for the Surface of a Spherical Probe with r = 0.5 cm (1), 1 cm (2) and 2 cm (3).

In the practical utilization of the spherical probe to determine the thermo-physical characteristics of natural soils it is usually necessary to deal with the difficulty of installing the probe in the soil. The spherical shape of the probe does not allow its installation without significant disruption of the natural structure of the soil. This makes it possible to use spherical probes for all practical purposes only for loose soil (sand). The recommendations of the authors regarding the timely installation of the probes in order that the structure of the soil will have a change to recover when the measurements are made, is not very effective and cannot always be accomplished. In this regard, the cylindrical shape of the probe has unquestioned advantages.

To determine the thermal conductivity of the soil by means of a cylindrical probe, it is necessary to find the expression for the temperature of some point on the probe as a function of the power of the heater and the thermophysical characteristics of the medium (soil) in which the probe is located. After measuring the temperature and knowing the power of the heater, we will obtain an equation for determining the thermophysical characteristics of the medium. Mathematically, this problem may be formulated as follows. Given a long cylinder with a radius R with thermophysical characteristics k_1 and λ_1 (temperature conductivity and thermal conductivity). Within the cylinder, a heat source operates at a constant power q. The initial temperatures of the cylinder and the medium equal zero (we will find the excess of the temperature over its initial value). Then, for the distribution of the temperature in the cylinder (1) and the medium (2), we will obtain the following equations [13]:

$$\frac{\partial t_1(r,\tau)}{\partial \tau} = k_1\left[\frac{\partial^2 t_1(r,\tau)}{\partial r^2} + \frac{1}{r}\frac{\partial t_1(r,\tau)}{\partial r}\right] + \frac{q}{c_1\rho_1} \quad (0<r<R), \tag{6}$$

$$\frac{\partial t_2(r,\tau)}{\partial t} = k_2\left[\frac{\partial^2 t_2(r,\tau)}{\partial r^2} + \frac{1}{r}\frac{\partial t_2(r,\tau)}{\partial r}\right] \quad (R<r<\infty). \tag{7}$$

As the boundary conditions for solving these equations, we will use the following:

$$\frac{\partial t_1(0,\tau)}{\partial r} = 0, \quad t_1(0,\tau) \neq \infty, \tag{8}$$

$$\frac{\partial t_2(\infty, \tau)}{\partial r} = 0, \tag{9}$$

$$t_1(R, \tau) = t_2(R, \tau), \qquad \frac{\lambda_1}{\lambda_2}\frac{\partial t_1(R, \tau)}{\partial r} = \frac{\partial t_2(R, \tau)}{\partial r}. \tag{10}$$

The physical sense of these conditions is quite obvious. To solve the equations, let us use the familiar Laplace transform [14]. Then, instead of equations in partial derivatives for the distribution of the temperature in the cylinder and the soil, we will obtain ordinary differential equations for the representations of the temperature in the plane of a complex variable: /75

$$sT_1(r, s) = k_1\left[\frac{\partial^2 T_1(r, s)}{\partial r^2} + \frac{1}{r}\frac{\partial T_1(r, s)}{\partial r}\right] + \frac{q}{c_1\rho_1}, \tag{11}$$

$$sT_2(r, s) = k_2\left[\frac{\partial^2 T_2(r, s)}{\partial r^2} + \frac{1}{r}\frac{\partial T_2(r, s)}{\partial r}\right], \tag{12}$$

in which the complex variable s may be viewed as a parameter. The solutions to these equations are obtained in the following fashion:

$$T_1(r, s) = \frac{K_1 q}{\lambda_1 s^2}\left[1 - \frac{K_1\left(\sqrt{\frac{s}{K_2}}R\right) I_0\left(\sqrt{\frac{s}{K_1}}r\right)}{I_0\left(\sqrt{\frac{s}{K_1}}R\right) K_1\left(\sqrt{\frac{s}{K_2}}R\right) + \frac{\lambda_1}{\lambda_2} I_1\left(\sqrt{\frac{s}{K_1}}R\right) K_0\left(\sqrt{\frac{s}{K_2}}R\right)}\right]. \tag{13}$$

$$T_2(r, s) = \frac{k_1 q}{\lambda_2 s^2}\,\frac{I_1\left(\sqrt{\frac{s}{k_1}}R\right) K_0\left(\sqrt{\frac{s}{k_2}}r\right)}{I_0\left(\sqrt{\frac{s}{k_1}}R\right) K_1\left(\sqrt{\frac{s}{k_2}}R\right) + \frac{\lambda_1}{\lambda_2} I_1\left(\sqrt{\frac{s}{k_1}}R\right) K_0\left(\sqrt{\frac{s}{k_2}}R\right)} \tag{14}$$

where $I_\nu(z)$ and $K_\nu(z)$ are the modified Bessel functions of the first and second type of ν order. Shifting from the representation of the function T(r, 2), to its original $t(r, \tau)$, we will obtain for the surface of the probe

$$t(R, \tau) = \frac{4qR^2}{\lambda_2 \pi^2}\int_0^\infty \left(1 - e^{-\frac{K_1\tau}{R^2}\mu^2}\right)\frac{I_0(\mu)\, I_1(\mu)\, d\mu}{\mu^4\left[\varphi^2(\mu) + \psi^2(\mu)\right]}, \tag{15}$$

where

$$\varphi(\mu)=\frac{\lambda_1}{\lambda_2}I_1(\mu)\,Y_0\left(\sqrt{\frac{K_1}{K_2}}\,\mu\right)-I_0(\mu)\,Y_1\left(\sqrt{\frac{K_1}{K_2}}\,\mu\right),$$

$$\psi(\mu)=\frac{\lambda_1}{\lambda_2}I_1(\mu)\,I_0\left(\sqrt{\frac{K_1}{K_2}}\,\mu\right)-I_0(\mu)I_1\left(\sqrt{\frac{K_1}{K_2}}\,\mu\right).$$

To determine the thermal conductivity λ_2 and the temperature conductivity of the soil K_2 according to this formula, it is necessary to measure the increase in the surface temperature of the probe at two different times. We will obtain a system of two equations with two unknowns, whose solution gives us the desired values K_2 and λ_2.

Sufficiently good results in the determination of thermophysical characteristics of the soil may be obtained by using a simplified formula for the temperature of the surface of the cylindrical probe. This formula is obtained from equation (15) with several simplifying assumptions, the most important of which is the disregard of the thermal capacity of the probe. This solution, in the form

$$T(\tau)=\frac{q}{2\pi\lambda}\left[-Ei\left(-\sqrt{\frac{R}{2\sqrt{K\tau}}}\right)\right] \tag{16}$$

was used by A. D. Maysener [15]. Determination of the thermal conductivity and the temperature conductivity according to this formula is carried out similarly to their determination by means of a spherical probe using formula (2). Here again, the ratio $T(\tau_1)/T(\tau_2)$ depends only on K, which is also determined from the previously plotted graph for the given probe. Having /76 determined K, we can determine λ as well from the equation for $T(\tau_1)$ or for $T(\tau_2)$.

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*Similarly, in formulas (13) and (15) K_1 and K_2 in the argument should be k_1 and k_2. In formula 15, $\phi(\mu)$ and $\psi(\mu)$ of the sign of the Bessel function $I'(\mu)$ should everywhere read: $I(\mu)$ with appropriate subscript.

A considerable increase in the accuracy of determination of thermophysical characteristics of soil according to formulas (2) and (8) may be achieved if we use the value q to calculate the quantity of heat which is expended in raising the temperature of the probe. This quantity of heat may be calculated easily from the known thermal capacity of the probe and the increase in the temperature of its surface.

Preliminary results obtained with a AFI spherical probe, as well as with models of cylindrical probes, indicate that with a cylindrical probe it is possible to ensure an accuracy of measurement of thermosphysical characteristics of soil of two to three percent with a measurement time of 10 to 15 minutes.

Mention should also be made of the theoretical possibility of using probes to determine the thermophysical characteristics of media in which heating causes phenomena that distort the values of thermal conductivity and temperature conductivity (for example, thawing of frozen soil, thermal convection in fluids, etc.). These possibilities were tested experimentally jointly with D. L. Laykhtman in determining the molecular thermal conductivity and temperature conductivity of water. To do this, the distorted values of K' and λ' of water were determined at different degrees of heating of the heater (due to changes in the intensity of the current in the latter) and the results were extrapolated for a zero value of heating. Then the relationship of K' and λ' in the system of coordinates K - ΔT and ln λ - Q is quite well incorporated in a linear relationship (Figures 3 and 4). For other phenomena associated with the change in the thermophysical characteristics, these relationships will obviously be slightly different but their experimental determination will not pose any particular difficulty, thus making it possible to obtain sufficient accuracy /77
by extrapolation, especially since extrapolation is carried out over a comparatively narrow range ΔT (2-3°).

Hence, with a sufficiently reliable determination of the thermophysical characteristics of the soil the heat flux on the surface into the depths for any time interval may be calculated by formula (1). As we can see from this formula, the error in the value of the heat flux at the depth of the quasi-stationary layer will be composed of the error in measurement of λ and the and the error in the determination of $\partial T/\partial z$ (or $\Delta T/z_1 - z_2$). When using refined methods of determining thermophysical characteristics, the error in the determination of λ will not exceed 3 to 5°. As a result, the error in determining the heat flux due to the error in determination of λ will also be no more than 3 to 5%.

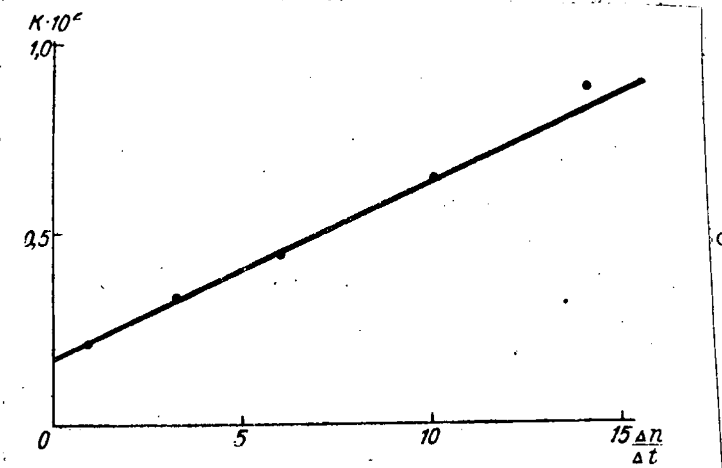

Figure 3. Temperature Conductivity of a Probe at Various Values of the Heater Temperature.

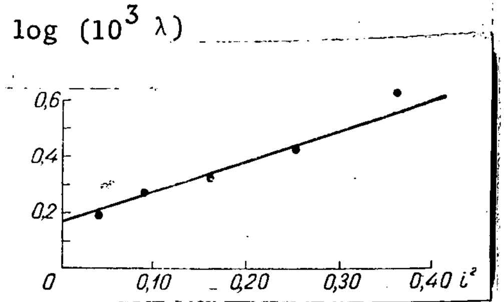

Figure 4. Thermal Conductivity of Water as a Function of Current Strength in a Probe Heater.

An error in measuring the temperature differential will produce a constant error in the value of the heat flux which will be independent of the heat flux value. In the Savinov elbow thermometers used in the network for measuring soil temperature, the magnitude of this error amounts to 0.04 λ on the average, which at average heat flux values in the soil at a depth of the quasistationary layer for middle latitudes is on the order of $5 \cdot 10^{-4}$ calories per cm^2/sec is approximately 25%. At low heat flux values ($1.5 \cdot 10^{-4}$ calories per cm^2/sec) even the sign of the heat flux may be incorrect.

Increasing the accuracy of soil temperature measurements likewise makes it possible to decrease the magnitude of this constant error. Thus, the use of remote electric thermometers for these purposes, which ensure a temperature measurement accuracy of 0.1° [16], obviously makes it possible to reduce this error from 0.040 to 0.015 λ.

Determination of the change in the heat content of the soil layer usually does not pose difficulties, especially since the volume heat capacity can be determined as the ratio of the fraction λ/K or can be determined by the nature of the soil. Due to the fact that this portion of the heat flux is usually small in absolute value (the thickness of the layer of nonstationary thermal regime is usually small), the thermal capacity can be determined with low accuracy.

Regardless of the possibilities presented here for increasing the accuracy of determination of the heat fluxes in the soil and the simplification of the achievement of the necessary measurements and calculations, this method of determining heat exchange between the surface and deep layers of the soil is still laborious. Considerable possibilities in this regard are opened up by direct methods of heat flux measurement in soil.

Recently, primarily in techniques for the direct measurement of heat flux in solids, so-called heat-meters have begun to be used. Heat-meters are plates, disks, or strips (depending on their specific application) made of a material with a known thermal conductivity. In a quasistationary regime, the heat flux through the body of the heat-meter, directed perpendicular to its surface may be written with sufficient accuracy in the form

$$Q' = -\lambda' \frac{\Delta T}{h}, \tag{17}$$

where h is the thickness and λ' is the thermal conductivity of the sheet of the heat-meter, ΔT is the temperature differential on its surfaces. For measuring the temperature differential on the surfaces of the heat-meter, a battery of thermocouples is usually employed; several of their junctions are located on the upper surface while others are on the lower. /78

The basic problem with using heat meters for measuring heat fluxes is the determination of the ratio between the heat flux through the heat meter and the heat flux in the surrounding medium. The first attempts to use heat meters were based on a simple assumption that the heat flux through the heat meter is equal to the heat flux in the medium. However, this assumption obviously is valid only when the thermophysical characteristics of the heat meter and the medium are the same. In the general case, the heat flux through the heat meter is proportional to the heat flux in the medium in which it is located, so that the coefficient of proportionality depends on the size of the heat meter and the thermophysical characteristics of the medium and the heat meter.

Taking into account the difference in heat flux through the heat meter and the heat flux in the medium in which it is located, G. A. Al'perovich [17] proposed a special method for calibrating heat meters.

This method consists in measuring the temperature differential on the surfaces of the heat meter with fixed values of the stationary heat flux developed in a special apparatus. The differences in thermophysical characteristics of the medium in which the heat meter is calibrated and of the medium in which it will be used are essentially disregarded. In technical measurements this is permissible, but for our purposes such an allowance would be excessively rough.

An attempt to obtain an analytical relationship between the heat flux in the soil and the heat flux through the heat meter, located in it, at a certain depth undertaken by A. G. Kolesnikov and A. A. Speranskaya [18] also did not meet with success. Excessively generalized equations, used as the basis for determination of the temperature field near the heat meter led to a rough estimate and an incorrect conclusion that the heat flux through the heat meter is practically equal to the heat flux in the soil at any values of the thermal conductivity of the soil and heat meter.

For a smoother interpretation of the physical picture of the propagation of heat in soil in which a heat meter is installed, it is possible to use the basic concepts of mathematical field theory. For the sake of simplicity of calculations, we shall assume that the soil constitutes a homogeneous medium (in the thermophysical sense) with a thermal conductivity λ, and with the heat flux in the soil being stationary, while a heat meter with a thermal conductivity λ' is mounted at a distance from the surface such that the variations in the temperature field at the surface of the soil will be negligibly small.

The latter condition makes it possible to view the soil as an unbounded medium without internal sources in which a stationary heat flux is assumed. In addition, for the sake of simplicity we shall view the plane problem when the dimensions of the heat meter are infinite in one direction that is perpendicular to the heat flux. This restriction is quite serious, but the results obtained may be extended without considerable difficulty to bodies in the shape of a disk, etc. with a transition to a cylindrical system of coordinates.

By locating the origin of the coordinates in the center of the heat meter and directing axis x along the normal to this surface, we will obtain for the established plane flux without sources:

$$\left.\begin{aligned} \operatorname{div} Q &= \frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} = 0 \\ \operatorname{Curl} Q &= \frac{\partial Q_y}{\partial x} - \frac{\partial Q_x}{\partial y} = 0 \end{aligned}\right\} \qquad (18)$$

i.e., the usual equations for potential flow. The role of the potential is /79
played here by the temperature T:

$$Q = -\lambda \operatorname{grad} T, \quad Q_x = -\lambda \frac{\partial T}{\partial x}, \quad Q_y = -\lambda \frac{\partial T}{\partial y}. \qquad (19)$$

From (18) and (19) we can conclude that the expression $1/\lambda\ (Q_y\, dx - Q_x dy)$ is the complete differential of some function U = U(x, y), which is completely analogous to the current function in hydromechanics. U(x, y) is linked to temperature by the Koshi-Riemann conditions:

$$\left.\begin{aligned} \frac{\partial U}{\partial x} &= \frac{\partial T}{\partial y} = -\frac{1}{\lambda} Q_y \\ \frac{\partial U}{\partial y} &= \frac{\partial T}{\partial x} = \frac{1}{\lambda} Q_x \end{aligned}\right\} \qquad (20)$$

The lines on which U(x, y) assumes constant values may be viewed as heat flux lines [19]. Then, by analogy with the equations of hydromechanics for flow around a flat disk of finite dimensions by a current of ideal fluid [20], the solution of the equations for flow around the disk of the heat meter by a heat flux also gives an exponential relationship for the temperature distribution near the heat meter with respect to the distance from its center. Then, instead of the condition of equality to zero of the fluid flux through the solid disk, we use the condition of equality of the normal components of the heat flux in a medium and in a heat meter at the interface:

$$-\lambda \frac{\partial T}{\partial x} = -\lambda' \frac{\partial T'}{\partial x}.$$

For the heat flux through the heat meter, we will obtain the ratio in the following form:

$$Q' = Qe^{a\left(1-\frac{\lambda'}{\lambda}\right)}, \tag{21}$$

where a is a constant. The dependence of the conversion factor for the heat meter $A = Q/Q'$ is obtained in the form

$$A = e^{a\left(\frac{\lambda'}{\lambda}-1\right)}. \tag{22}$$

As we can see from (22), if $\lambda' = \lambda$, $A = 1$ and the heat flux through the heat meter is equal to the heat flux in the soil.

It should be pointed out that a similar relationship of the conversion factor for the heat meter was assumed on the basis of purely qualitative considerations by D. Portman [22].

The data presented in [22] on the generalization of experimental data is in satisfactory agreement with the form of the relationship adopted.

Taking the logarithm and differentiating expression (22), we will have

$$\frac{\partial A}{A} = a\,\frac{\lambda' d\lambda}{\lambda^2}. \tag{23}$$

Usually the value a is less than one (under the condition that the width of the heat meter is considerably greater than the thickness); if $\lambda' < \lambda$, the relative change in the conversion factor will be less than the relative change in the thermal conductivity of the soil, i.e., within certain limits of variation of the thermal conductivity of the soil the conversion factor for /80 the heat meter may be considered roughly constant.

This has been used as a basis for the suggested method of using the heat meter to measure heat fluxes in soil.

As is indicated by analysis of the data from a measurement of the thermophysical characteristics of soil, the latter change comparatively slowly. Abrupt changes have an episodic nature and are associated with periods of excessive rain and other phenomena that are seasonal as a rule.

The determination of A can be carried out in practice more conveniently by measuring Q and Q'.

To calculate Q, we can use the method described above for measuring the thermophysical characteristics of the soil and the temperature distribution in it. Then, for a reliable determination of A, it is sufficient to have three or four values of Q. To shorten the calculations, it is convenient to use instead of A the coefficient $A' = -\lambda' A/h$. Then $Q = A'\Delta T$. Figure 5 shows the heat flux curve in soil according to heat meter data for the 23-24 August 1958 at the MGO station in Koltushiye. This heat meter was a flat ring 1 cm thick made of plexiglas. The even junctions of a thermopile were uniformly attached to the upper surface of the heat meter, while the odd junctions were located on the lower surface. The thermopile consisted of 20 pairs of junctions made of rolled manganin-constantan strip, 0.05 mm thick. To measure the heat flow in this battery, a M-91 microammeter with a R-4 universal shunt was used. When set to maximum sensitivity, one division on the microammeter scale corresponded to 0.007°. Subsequently the microammeter was replaced by a less accurate moving magnetic needle galvanometer (M-117/3), which does not require shunting at high heat flux values.

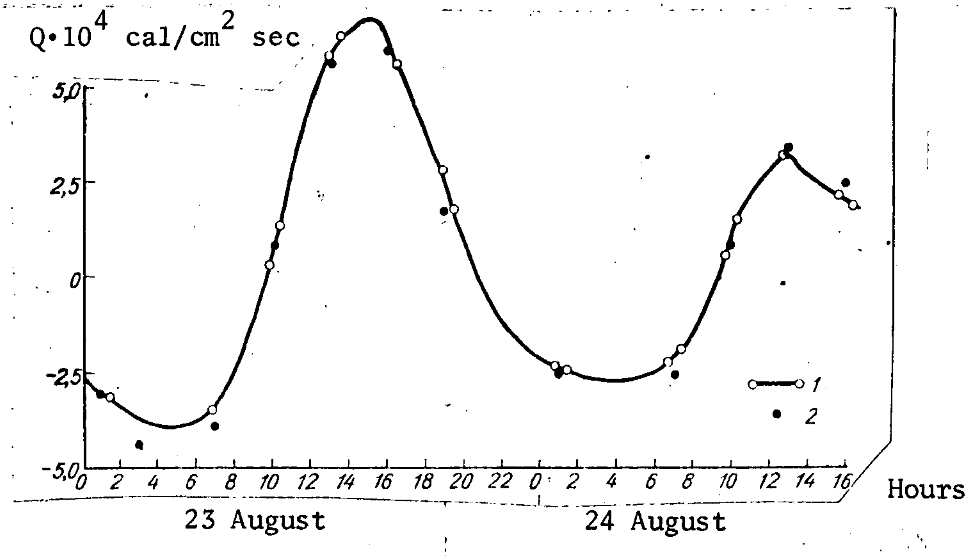

Figure 5. Heat Flux as Indicated by a Heat Meter (1) and Calculated Values of Heat Flux (2) for the 23-24 August.

The control values for heat flux obtained from the data on thermal conductivity of the soil and the temperature distribution in it are marked with an asterisk (*) [2]. As we can see from Figure 6, the differences lie within the limits of accuracy of the control method. The value of the coefficient A was found to be equal to $0.587 \cdot 10^{-4}$ cal/cm^{2}sec·deg. /81

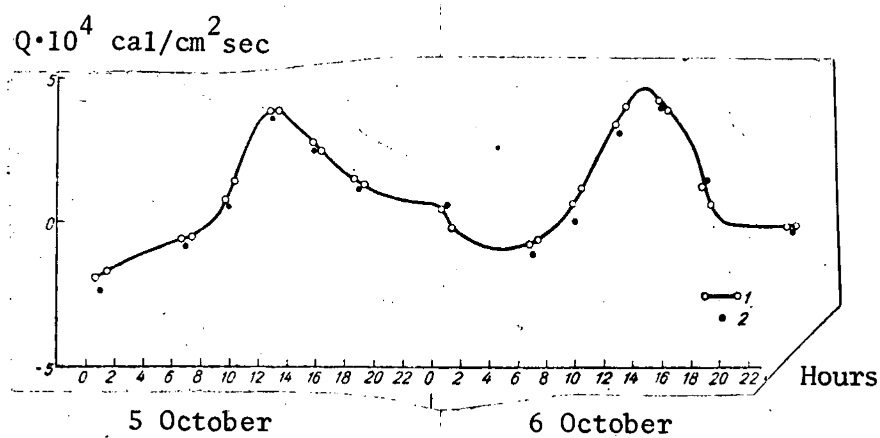

Figure 6. Heat Flux as Indicated by Heat Meter (1) and Calculated Values for Heat Flux (2), 5-6 October.

Figure 6 shows the curve of the change in heat flux in the soil as indicated by the same heat meter for the 5th-6th October 1958. The heat flux values were calculated with the same value of A. As we can see from the distribution of the control values for the heat flux, in this case also the deviations lie within the limits of accuracy of the control method. An analysis of the data for all intermediate days shows the same agreement. Consequently, during the period from August through September the value of the conversion factor remained constant, regardless of certain variations in the thermal conductivity of the soil (from 0.0025 to 0.0030 cal/cm^{2}sec·deg, i.e., by 20%).

In September 1959, this heat meter was used to measure heat flux in conjunction with the work of the joint expedition to study the structure of the layer of the atmosphere closest to the ground in southern Kazakhstan (Kyzyl-Kumy). As we expected, the conversion factor for the heat meter A remained constant during the entire working period of the expedition. According to data from direct measurements, the thermal conductivity of the soil at the point of installation of the heat meter was 0.00082 cal/cm^{2}sec·deg. The conversion factor for the heat meter A' was found to be equal to $0.440 \cdot 10^{-4}$ cal/cm^{2}sec·deg. Hence, when the thermal conductivity of the soil changed by a factor of three or a little more.

As an example of the observed pattern for heat flux as indicated by the heat meter on this expedition, we have plotted in Figure 7 the results of measurements for the 16-17 September 1959 (Curve 2). In this figure (Curve 1) we have also included the results of heat flux measurements as indicated by a heat meter with lesser thickness (0.6 cm). The somewhat greater thermal capacity of the heat meter 1 cm thick leads to a slight delay in the phase of the corresponding values of the heat flux. This figure also clearly shows the slight delay in the heat flux values obtained on the basis of the thermometer data. This may also be explained by the significant thermal capacity of the metal resistance thermometers (we are referring to the thermal capacity relative to a unit volume). In this case, this delay in time is comparatively small and is not of significant importance. However, if we take into account the considerably greater mass of the soil extraction thermometers used in the network we can conclude that their delay is severalfold greater. It should also be pointed out that there is a good agreement of the data from both heat meters, which is due to a certain degree to an additional averaging of the /82 heat flux over the area.

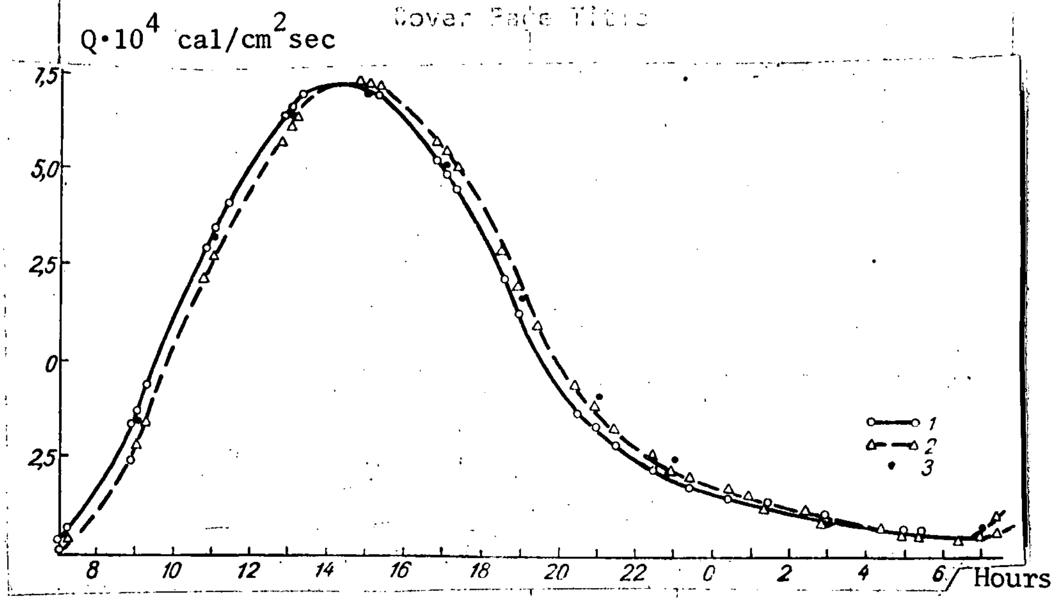

Figure 7. Heat Flux as Indicated by Heat Meter h = 0.6 cm (1), h = 1.0 cm (2) and Calculated Heat Flux Values (3).

In conjunction with the error in measurement of heat flux in the soil, the heat meter (in comparison with calculation methods) has the unquestioned advantage that the constant error due to the error in measurement of temperature differential on its surfaces may be reduced to a negligibly small value by appropriate calculations of the sensitivity of the thermopile. In particular, the heat meter which we used in Koltushaya allowed measurement of temperature differential on the surfaces with an accuracy up to 0.002°, which corresponds to the error in the heat flux value through the heat meter of 0.002 λ'. However, the error due to change of conversion factor with decreasing heat flux value decreases.

Thus, the control measurements of the conversion factor may be performed episodically, selecting times when the heat flux in the soil is sufficiently high. Then the error in the calculation method will be less.

By means of the heat meter, it is easy to achieve automatic recording of heat flux in the soil. To do this, it is merely necessary to use instead of the galvanometer a sufficiently sensitive recording electric measuring device (for example, the EPP-09 or galvanograph).

Hence, the use of a heat meter to measure heat flux in the soil considerably reduces the laboriousness of determining heat flux, reducing all operations to reading of a galvanometer and multiplying it by a conversion factor. The conversion factor can be monitored approximately once a month, selecting days when the heat flux value is sufficiently high.

To check the conversion factor, the heat flux in the soil should be determined as the derivative of the thermal conductivity of the soil over the temperature gradient in it. Then, to measure the temperature distribution in /84
the soil, it is necessary to use resistant thermometers which will allow sufficient accuracy of measurement and will have a comparatively low thermal capacity.

This method may be recommended for extensive utilization at hydrometeorological stations.

TABLE 1.

Date	Ia	ΔT_2	ΔT_6	$\frac{\Delta T_2}{\Delta T_6}$	$K \cdot 10^3$	$1 - \Phi\left(\frac{2}{2\sqrt{120K}}\right)$	$1 - \Phi\left(\frac{2}{2\sqrt{360K}}\right)$	$\lambda \cdot 10^3$ cal/cm^{2}sec·deg. 2 min.	$\lambda \cdot 10^3$ cal/cm^{2}sec·deg. 6 min.
5 cm Deep									
6 June	0,10	2,71	3,32	0,817	7,88	0,644	0,790	1,66	1,65
18 June	0,10	1,44	1,78	0,808	7,20	0,629	0,781	1,34	1,34
	0,15	3,16	3,93	0,804	6,96	0,624	0,778	1,38	1,39
19 June	0,10	1,44	1,80	1,798	6,56	0,614	0,772	1,32	1,33
21 June	0,15	3,14	3,89	0,807	7,16	0,628	0,780	1,40	1,40
20 cm Deep									
6 June	0,10	0,96	1,31	0,738	4,16	0,527	0,714	1,70	1,69
18 June	0,10	0,96	1,31	0,738	4,16	0,527	0,714	1,70	1,69
	0,15	2,29	2,91	0,787	5,96	0,596	0,760	1,82	1,83
19 June	0,10	0,97	1,31	0,745	4,32	0,537	0,720	1,72	1,71
21 June	0,15	2,20	2,95	0,747	4,24	0,531	0,717	1,69	1,70

Note: Commas indicate decimal points.

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